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Analysis of Mathematics Education Students' Errors in Solving Limit Function Problems

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Abstract

This study aims to describe students' errors when solving limit function problems in the Calculus I course. This research is a qualitative descriptive study. The subjects in this study were taken from mathematics education students at a university in Riau Kepulauan, Indonesia. The methods of collecting the data used in this study were tests and interviews. Before the researcher conducted the analysis, the researcher examined the validity of the data using triangulation between researchers, where the method used is more than a researcher in analyzing and collecting data to obtain valid data. Furthermore, the valid data was analyzed and concluded. From the results of the research, it was found that students could understand the facts presented, but most students made errors in solving the problems presented with a concept error of 32.35% (low category), principle errors of 29.41% (low category), and operating errors of 41.18% (medium category). Problem-solving errors occur because of wrong choices of the true solution and a lack of rigorous students in the completion of them. So, for further research, it is necessary to examine what factors cause students to be less thorough in solving math problems.

Keywords: analysis, calculus 1, errors, limit function

INTRODUCTION

Calculus (Latin: calculus, meaning "small stone", for counting) is a branch of mathematics that includes limit, derivative, integral, and infinite series. Calculus is the study of change, as geometry is the study of form and algebra is the study of work to solve equations and their applications. Calculus has wide applications in the fields of science, economics, and engineering (Fuentealba *et al.*, 2019), and can solve various problems that cannot be solved with elementary algebra. Calculus has two main branches: differential calculus and integral calculus, which are interconnected through the basic theorem of calculus (Rahman, 2019). Calculus learning is the gateway to other higher mathematics lessons, which specifically study functions and limits, which is generally called mathematics analysis.

Calculus I is a compulsory subject in the program study of mathematics education at a university in Riau Kepulauan, Indonesia with a charge of 3 credits. The material is in the form of a real number system, inequality, inequality and absolute value, one variable function, types of functions, operations in functions, composition functions, inverse functions, implicit functions, trigonometric functions, cyclometric functions, graphs functions, limits functions, continuity functions, limit function theorem, continuous function, counting limits function, derivative function and theorems, definition of derivative geometry function, continuity and differentiation, chain rule, implicit differentiation, differential and derivative, application of derivative function, drawing graph function, and derivative function in some problems.

Calculus I is an important basic course to be mastered by students because it is widely used to study other subjects (Gerhard *et al.*, 2015). Therefore, this subject is a prerequisite for taking the next few courses. Based on the researcher's experience when teaching

Calculus I subjects, they found errors in solving problems on the Calculus I course in the form of conceptual or non-conceptual errors, principle errors, and operating errors had an impact on the low student learning outcomes. Based on the list of academic scores of mathematics education students in Calculus I, there were still many students who scored below 68 (category C). Students' obstacles to learning basic calculus in general lie in the fundamental abilities of algebraic functions and limit functions. There are student errors due to carelessness, errors in processing skills, errors in understanding questions, errors in transformation, and errors in using notation (Wahyuni, 2017; Jaafar & Lin, 2017).

Error is a form of deviation from actual answers that are systematic in nature (Wahyuni, 2017). Error analysis is an attempt to observe, discover, and classify errors with certain rules (Ardiawan, 2015). According to Siswandi & Sujadi (2016) and Astuty & Wijayanti (2013), students' errors need to be analyzed to find out the various errors made by students. Through this analysis, the type and location of errors will be determined, so that educators can provide the right solution so that it can be improved and not repeated. Information errors in solving math problems can be used to increase the effectiveness of mathematics learning.

Hidayat classifies errors in four types, namely, factual errors, concepts, principles and operating errors (Widodo & Sujadi, 2015). Factual errors mean students are not able to convey the material in the problem such as incorrectly changing the problems that exist in the problem into the mathematical model, as well as errors in writing mathematical symbols; Errors concept consist of a) students incorrectly use the concept of variables used, b) the use of formulas, theorems or definitions do not adjust to the prerequisite conditions of entry into force, and c) students did not write formulas, theorems or definitions to answer the problem; The principle errors mean students misinterpreted the questions; Operating errors consist of a) errors in calculating and b) students were not able to manipulate steps to answer a problem.

Research on error analysis has also been done, including analysis of student errors in solving inequality problems in Calculus I (Rahmawati, 2017); error analysis in solving mathematical problems (Istiqomah, 2016); solving mathematical induction problems; in solving the problem of type divergence proves (Widodo, 2013); and the analysis of student errors in the prerequisite 1 calculus courses has also been done by Abidin (2012) in trigonometric problems.

Based on the description of previous explanations and research, there is a need for an analysis of student errors in completing limit function questions in the Calculus I course to be able to find out and identify and describe more clearly what mistakes are made by students. Besides that, this study will also explore why students made errors so that the teachers know why students make mistakes and can improve through learning.

METHOD

This research is a qualitative descriptive study. This study aims to describe student errors in completing limit function problems in Calculus I. The subjects in this study were taken from mathematics education students at a university in Riau Kepulauan, Indonesia, which consisted of 34 students in the first semester with the following criteria: Factual errors, principle errors, conceptual errors, and operating errors (see Table 1).

Table 1. Criteria for Error Analysis of Problem Solving

Interval	Category
0-20	Very low
21-40	Low
41-60	Medium
61-80	High
81-100	Very high

(Sigit, Ernawati, & Qibtiah, 2017)

The main instrument in this research is the researchers themselves because, in this study the researchers are the determinant in collecting, analyzing, and presenting data.

While the minor instruments in this study are limit function test questions and interview guidelines. The test instrument used is valid by using content validity and reliability (r_{11} = 0.722). The data collection techniques used in this study were tests and interviews. The test used the form of giving questions in the form of descriptions related to the limit function material given to all mathematics education students who took calculus I. The test was given after obtaining limit function material. The interview in this study is conducted by interviewing the subject based on the written results of the questions given, aiming to clarify the written answers to the subject and to obtain information on why students made errors. Interview guidelines are unstructured because researchers do not use interview guidelines that have been systematically and completely arranged to collect data, but rather the interview guidelines used are only outlines of the problems to be asked (Sugiyono, 2016). The question given by the researcher does not have to be the same for each subject but depends on the amount of information needed by the researcher. Before the researcher conducted the analysis, they examined the validity of the data using triangulation between research methods where the method used is more than a researcher in analyzing and collecting data to obtain valid data. Subsequently, the new valid data was analyzed and then a conclusion was drawn.

RESULTS AND DISCUSSION

The results of the percentage analysis of student errors in completing the limit function problems during the final exam taken from 34 students are presented in Table 2. The average score can be seen that the whole student can understand what facts are presented. Hence the level of error category is very low, and the biggest error occurs in operating errors in the medium category. In more detail, the biggest error of 55.88% occurs in conceptual errors in question number 2. Furthermore, the level of analysis of students' errors based on the type of error is shown in Table 3.

Table 2. Errors percentage made based on Type of Error

Category		Result	Average	Category
Factual	1	0.00	0.00	Very low
	2	0.00		
Concept	1	8.82	32.35	Low
	2	55.88		
Principles	1	35.29	29.41	Low
	2	23.53		
Operation	1	35.29	41.18	Medium
	2	47.06		

Concept Error Type

In the type of error concept, there were 3 people in question number 1 (8.82%). The error made by the student was choosing the completion technique presented, where students completed using the wrong factors. This error can be seen in Figure 1a.

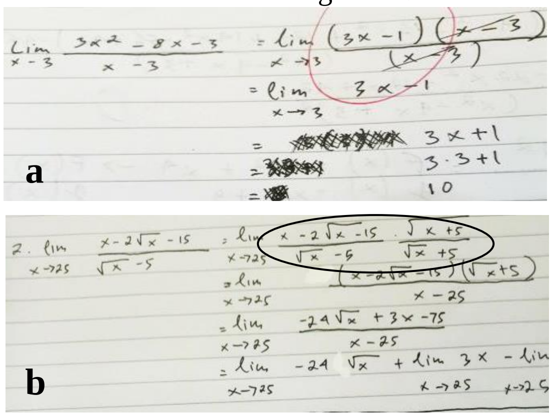

Figure 1. Type of Error Problem Number 1

In Figure 1, students are mistaken in factoring choices, should be a factor of $3x^2 - 8x - 3 = (3x+1)(x-3)$ not $3x^2 - 8x - 3 \neq (3x-1)(x-3)$. After conducting a brief interview to confirm the completion of the students.

Teacher: "*Why did you choose the factor from this equation?*"
Student: "*Because the factor of the equation is…*"
Teacher: "*Try to multiply again? Is the answer the same?*"
Student: "*Oh. Sorry mam, it turns out I was wrong here.*"

It was found that students were not careful in choosing the appropriate factors for the equation presented.

Table 3. Results of Students' Error Analysis for Each Item Based on the Type of Error Made

Question	Error Type			
	Factual	Concept	Principle	Operating
Decay value of the resistance of a building using a function $\frac{3x^2-8x-3}{x-3}$ Calculate the resilience value of the building by using the limit on x - 3	-	• Solve problems using the wrong factors	• Problem-solving using incompatible theorems	• An error occurs in the operation of the stage $\lim_{x\to 3}\frac{(3x+1)(x-3)}{x-3}$ to $\lim_{x\to 3}(3x-1)$
A dam has a water pressure resistance based on function $\frac{x-2\sqrt{x}-15}{\sqrt{x}-5}$ By using limit function, determine the value of the benefits when value $x-25$?	-	• Solve the problem by choosing an improper settlement technique, which should use the factoring method instead of equating with the root of peerage	• Problem-solving using an incompatible theorem	• An error occurs in the operation of the stage $\lim_{x\to 25}\frac{(x-2\sqrt{x}-15)(\sqrt{x}+15)}{x-25}$ to $\lim_{x\to 25}\frac{\sqrt{x}+3x-75}{x-25}$ • An error occurs in the operation of the stage $\lim_{x\to 25}\frac{(x-2\sqrt{x}-15)}{(\sqrt{x}-15)}\cdot\frac{(\sqrt{x}+15)}{(\sqrt{x}+15)}$ to $\lim_{x\to 25}\frac{-24\sqrt{x}+3x-75}{x-25}$ An error occurs in the operation $\frac{\sqrt{25}+3\,(75)-75)}{0}=5+3$

Furthermore, for question number 2, students made a conceptual error of 19 people (55.88%). This error can look like Figure 1b. In this problem, students incorrectly choose the method of solving the problem given. They solve the problem by using the method of multiplying the root of the student, which should use the factoring method such as Figure 2.

$$\lim_{x\to 25}\frac{(x--2\sqrt{x}-15)}{(\sqrt{x}-15)}$$
$$=\lim_{x\to 25}\frac{(x-2\sqrt{x}-15)}{(\sqrt{x}-15)}\cdot\frac{(\sqrt{x}+15)}{(\sqrt{x}+15)}$$

Wrong Solution

$$\lim_{x\to 25}\frac{(x--2\sqrt{x}-15)}{(\sqrt{x}-15)}$$
$$=\lim_{x\to 25}\frac{(\sqrt{x}-5)(\sqrt{x}+3)}{(\sqrt{x}-15)}$$

True Solution

Figure 2. Problem Solving to Number 2

To confirm the mistakes made by students such as in Figure 2, interviews were conducted where it was found that students chose the wrong completion technique.

Teacher: "*Why did you choose this way of solving equation?*"

Student: "*Because the equation can be solved using group root. This is because of the denominator of the problem.*"

Teacher: "*Can't the numerator of the equation be factored?*"

Student: "*Really, hm. I don't master it.*"

For the interviews, they can be obtained because there are roots in the denominator of the problem. With the root, which makes students choose the solution using group root.

Principle Error

In the type of principle error, there were 12 people in question number 1 (35.29%). The errors made by students were in the form of the wrong completion by using inappropriate theorems, which should have used factoring methods. The same thing happened in number 2, which was done by eight people (23.53%). This error can be seen in Figure 3.

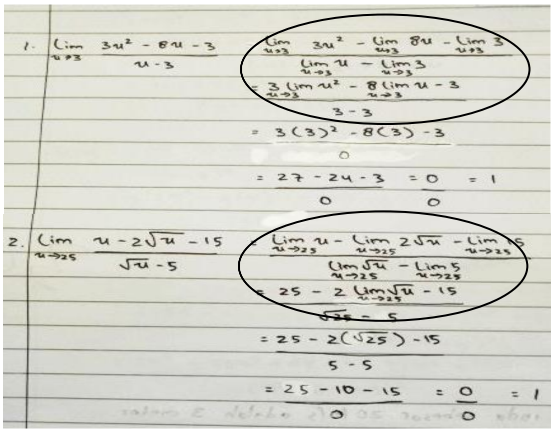

Figure 3. Type of Principle Error

From Figure 3, and after confirming with the students, they stated they were wrong in understanding the principle of solving the problem presented. The following is a conversation during an interview with students.

Teacher: "*Why did you choose this way of solving equation?*"

Student: "*I use the limit theorems to solve the equation.*"

Teacher: "*Doesn't the theorem not apply to this situation? You should use the factoring method.*"

Student: "*Sorry mam, I didn't even think about it.*"

The information from the interviews suggests that they still understand the solution using the existing limit theorems so they solve the problem using that principle. Where it is supposed to solve the problem presented using a factoring method.

Type of Operation Error

In the type of operation error, 12 people (35.29%) made errors on the number 1 problem. The error occurred in the operation from this stage:

$$\lim_{x\to 3}\frac{(3x+1)(x-3)}{x-3}=\lim_{x\to 3}(3x-1)$$

This error can be seen in Figure 4.

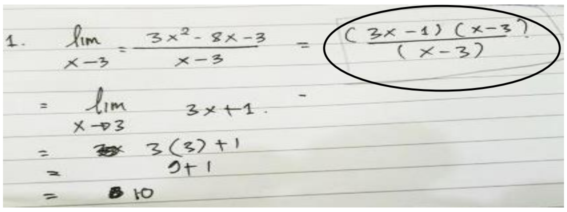

Figure 4. Type of Calculation Error Problem Number 1

Based on Figure 4, the students' calculation uses the $\lim_{x\to 3}(3x-1)$ .In addition, some students also write $\lim_{x\to 3}$ while the limit value has been included in the variable x. An interview was conducted to confirm the error.

Teacher: "*Why did you write the limit to the end?*"

Student: "*Because the topic studied is about limit, so in solving equations the limit must be written*"

Teacher: "*Isn't that when the value of x has been substituted, the limit should not be written again?*"

Student: "*Oh, I'm having fun writing it without paying attention to it.*"

The information was conducted and obtained from information that students wrote $\lim_{x\to 3}$ until the end of the score carelessly and were not careful in writing the mark where it should be $(3x+1)$ but written $(3x-1)$ which caused the acquisition of the intended limit value. As for the type of operation error, 16 questions (47.06%) were carried out in question number 2. An error occurred in the calculation operation or operation error from this step

$$\lim_{x\to 25}\frac{\left(x-2\sqrt{x}-15\right)\left(\sqrt{x}+15\right)}{x-25} \ggg \lim_{x\to 25}\frac{\sqrt{x}+3x-75}{x-25}$$

The error occurred in the operation of the steps

$$\lim_{x\to 25}\frac{\left(x-2\sqrt{x}-15\right)}{\left(\sqrt{x}-15\right)}\cdot\frac{\left(\sqrt{x}+15\right)}{\left(\sqrt{x}+15\right)} \ggg \lim_{x\to 25}\frac{-24\sqrt{x}+3x-75}{x-25}$$

This operating error can be seen in Figure 1b and Figure 5.

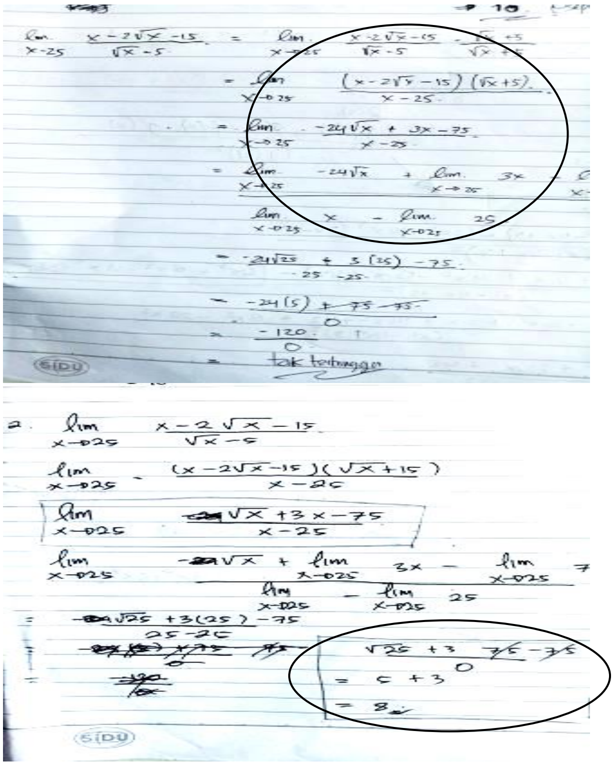

Figure 5. Type of Operation Error about Number 2

Based on the picture above, there are errors in the operations of students in solving the existing problems.

To make sure an interview

Teacher: "*Why did you divide a number by 0 to produce a number?*"

Student: "*Doesn't divide a number by 0 produce the number itself?*"

Teacher: "*Doesn't that return 0?*"

Student: "*Oh. I was wrong.*"

In the interview can get information, the entire student stated that he was not careful in calculating the process of completing the answers presented.

Based on the explanation of the data above, it can be seen that the errors that often occur in students when solving mathematical problems are errors in concepts, principles, and procedures. In the concept error of 32.35% (Low), students are less careful in choosing the appropriate factor for the equation presented and are wrong in drawing the square root, where students should make a solution with group roots. The principle error is 29.41% (Low), but they still understand the solution using the existing limit theorem, so they solve the problem using that principle. Where it should be to solve the problem presented using the factoring method. For procedural errors of 41.18% (Medium), he was less thorough in calculating the process of filling out the answers presented.

When viewed from the types of errors above, the biggest error is in procedural errors, where students are always careless in doing calculations to the end. This is in line with Himmi & Husna (2020), where the biggest errors occurred in students incorrectly calculating and writing the final answer. Hanifah (2021), where students in solving mathematical problems given are only limited enough to be able to solve them properly and correctly. Kepa & Ramli (2021) as for the causes of errors, among others, lack of mastery of prerequisite material, lack of understanding of basic concepts, and confusion about which parts must be modified to solve the problem.

CONCLUSION

Based on the results and discussion above, it was found that students could understand the facts presented, but most students made errors in solving problems presented with conceptual errors of 32.35% (low category), principle errors of 29.41% (low category) and operating errors of 41.18% (medium category). Problem-solving errors occur because of wrong choices, the true solution, and a lack of rigorous students. So, for further research, it is necessary to examine what factors cause students to be less thorough in solving math problems.

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